OPTIMIZATION OF CONDUIT HEATING

B. N. Tokarskii

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We investigate the problem of the change, with time, in the temperature of a heating medium, this change corresponding to the minimum duration of conduit heating under the condition that the thermal stresses in the wall do not exceed permissible limits.

When actuating powerful high-energy units with straightpass boilers, from the instant of burner ignition to the entry of the steam into the turbine, the rate of rise in the steam temperature is limited by the thermal stresses within the wall of the steam conduit. To reduce the duration of the actuation process and the consumption of fuel, it is advisable to maintain the stresses within the steam conduit at permissible levels throughout the entire heating period. Usually, the entire range of variation in T is broken down into 2-3 intervals, for each of which a permissible value of $dT/d\tau$ is specified. The latter value is set with consideration of the relationship between the permissible stresses and the temperature. It is assumed in the determination of the thermal stresses that there is a quasisteady temperature field within the wall of the steam conduit [1].

It follows from the data in [2] that to establish a temperature field that is close to the quasisteady, we need a sufficiently large interval of time that is commensurate with the duration of conduit heating. This conclusion is in agreement with the experimental data of [3]. Since the stresses in the prequisisteady regime are smaller than in the quasisteady regime, the latter curves of variation in T cannot be regarded as optimum.

For the case α = const, an approximate method is proposed in [2] for the plotting of the function $T = T(\tau)$ for which the maximum thermal stresses are constant throughout the entire heating period. Below we present the solution for the problem of optimizing the heating of a thick-walled conduit, in more general formulation: it is assumed that $\alpha \neq$ const and $|\sigma| \neq$ const.

In firing up a straightpass boiler an attempt is made to maintain specific relationships between temperature, pressure, and flow rate for the steam being produced, so that for α we can specify the relationship

$$\alpha = \alpha(T, t_1). \tag{1}$$

We will also assume that when $r = r_2$ there is no transfer of heat.

Comparison of σ_θ and σ_φ with the permissible values is accomplished for $r = r_1$ and $r = r_2$, where the thermal stresses attain their greatest values ($\sigma_{r_1} = \sigma_{r_2} = 0$). Since other loading factors (bending, pressure, etc.) are taken into consideration in the choice of $[\sigma]$, for the inside and outside surfaces the values of $[\sigma]$ are assumed to be different. The temperature difference across of the thickness of the wall is usually so small that the mechanical properties of the steel vary only slightly in the temperature range from t_1 to t_2. We can thus assume that

$$[\sigma]_1 = f_1(t_{av}),$$
$$[\sigma]_2 = f_2(t_{av}). \tag{2}$$

Assuming that β, E, and μ are functions of t_{av}, we can determine [4] the thermal stresses in an infinite hollow cylinder for $t = t(r)$ from the following formulas:

Polzunov Central Boiler Turbine Institute, Leningrad. Translated from Inzhenerno-Fizicheskii Zhurnal, Vo.16, No.3, pp.489-493, March, 1969. Original article submitted May 29, 1968.

$$\sigma_{\varphi 1} = \sigma_{\theta 1} = \frac{\beta E}{1-\mu}\,[-t(r_1) + t_{av}],$$
$$\sigma_{\varphi 2} = \sigma_{\theta 2} = \frac{\beta E}{1-\mu}\,[-t(r_2) + t_{av}]. \tag{3}$$

Since $\sigma_{\varphi 1} = \sigma_{\theta 1}$ and $\sigma_{\varphi 2} = \sigma_{\theta 2}$, for determinacy we will subsequently assume that $f_1(t_{av})$ and $f_2(t_{av})$ are the permissible values, respectively, for $\sigma_{\varphi 1}$ and $\sigma_{\varphi 2}$.

First let us consider the problem of plotting the function $q = q(\tau)$, for which one of the equations in (2) is satisfied.

Figure 1 shows the curves for the variation in the dimensionless thermal stresses when $q = 1$.

The distribution of temperature through the thickness of the wall – required for the determination of the stresses from formulas (3) – was determined by calculation with a Ural-2 digital computer, in accordance with a program developed at the Polzunov Central Boiler Turbine Institute. This program provides a solution for the differential heat-conduction equation by a finite-difference method involving application of the pivot method to solve the system of equations (see, for example, [1]).

It follows from the curves in Fig. 1 that unlike the case of $dT/d\tau = \text{const}$ when $q = \text{const}$ the state that is close to the quasisteady is established within a brief time interval that is 2-3 orders of magnitude smaller than the duration of the heating.

With a variable heat flow

$$\sigma_\varphi(r, \tau) = q(0)\,S(r, \tau) + \int_0^\tau \frac{dq}{d\tau_0}\,S(r, \tau - \tau_0)\,d\tau_0. \tag{4}$$

The integral in the right-hand member of (4) can be presented in the form of the sum of the integrals with limits from 0 to $\tau - \Delta\tau$ and from $\tau - \Delta\tau$ to τ. The value of $\Delta\tau$ is set small, but so that

$$\frac{S(r, \Delta\tau)}{S(r, \infty)} \approx 1. \tag{5}$$

When $\tau < \Delta\tau$ we will assume that $I(0, \tau - \Delta\tau) = 0$.

In turn, $I(0, \tau - \Delta\tau)$ can be presented in the form

$$I(0, \tau - \Delta\tau) = \sum_{j=1}^{j=n} I_j(\tau_j, \tau_{j+1}),$$

where $\tau_{n+1} = \tau - \Delta\tau$; $\tau_1 = 0$ and the values of τ_j are chosen so that when $\tau_j < \tau < \tau_{j+1}$ the quantity $dq/d\tau_0$ does not change sign.

Applying the theorem of the average and formula (5), we find that

$$I(0, \tau - \Delta\tau) \approx [q(\tau - \Delta\tau) - q(0)]\,S(r, \infty).$$

Analogously,

$$I(\tau - \Delta\tau, \tau) \approx \theta\,[q(\tau) - q(\tau - \Delta\tau)]\,S(r, \infty),$$

where $0 < \theta < 1$.

Thus, when $\tau > \Delta\tau$

$$\sigma_\varphi(r, \tau) = \{q(\tau - \Delta\tau) + \theta\,[q(\tau) - q(\tau - \Delta\tau)]\}\,S(r, \infty). \tag{6}$$

We will heat the conduit so that at any instant of time

$$q = \frac{[\sigma]_1}{S_{1\infty}}. \tag{7}$$

Within a small time interval $\Delta\tau$ the value of t_{av} varies insignificantly in comparison with the initial and final temperatures. Therefore, for the steels that are usually employed the increase in the permissible stresses during the period $\Delta\tau$ will be small in comparison with the magnitude of the stresses at the instant τ.

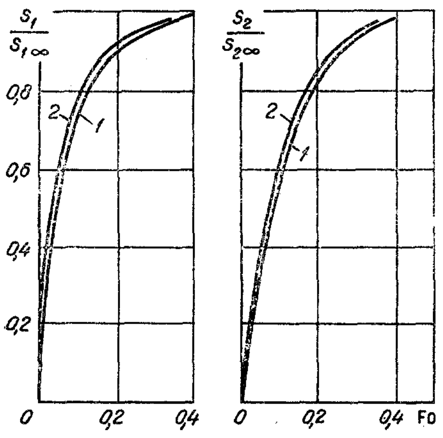

Fig. 1

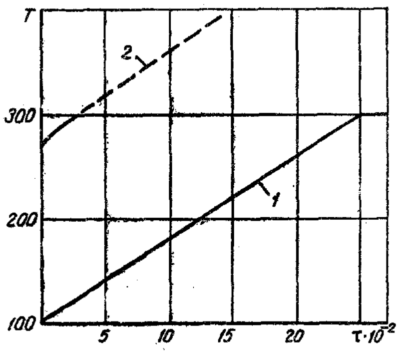

Fig. 2

Fig. 1. Stresses in a hollow cylinder at constant density for the heat flow through the inside surface: 1) for $r_2/r_1 = 1$; 2) for $r_2/r_1 = 2$.

Fig. 2. Curves for the change in the temperature of the heating steam (T in °C and τ in sec): 1) when $dT/d\tau = \text{const}$; 2) according to the proposed method.

Bearing this in mind, we find from (6) and (7) that

$$\sigma_{\varphi 1} \approx [\sigma]_1. \tag{8}$$

Analogously, when

$$q = \frac{[\sigma]_2}{S_{2\infty}} \tag{9}$$

$$\sigma_{\varphi 2} \approx [\sigma]_2. \tag{10}$$

On the basis of these results, we can recommend the following approximate method of plotting the function $T = T(\tau)$.

1. We determine the constant heating rate $dt_{av}/d\tau$ for the conduit in the case in which $q = 1$.

2. With the formulas for the determination [2] of the thermal stresses in the quasisteady regime

$$\sigma_{\varphi 1} = \frac{\beta E}{1-\mu}\,\frac{r_2^2}{8a}\left(3 - \frac{r_1^2}{r_2^2} + \frac{4r_2^2 \ln \frac{r_1}{r_2}}{r_2^2 - r_1^2}\right)\frac{dt_{av}}{d\tau},$$

$$\sigma_{\varphi 2} = \frac{\beta E}{1-\mu}\,\frac{r_2^2}{8a}\left(1 + \frac{r_1^2}{r_2^2} + \frac{4r_1^2 \ln \frac{r_1}{r_2}}{r_2^2 - r_1^2}\right)\frac{dt_{av}}{d\tau}$$

we calculate $S_{1\infty}$ and $S_{2\infty}$.

3. From formulas (7) and (9), and from the specified functions (2), we plot the curve for $q = q(t_{av})$.

4. With the aid of the results derived in item 3, we determine the function $t_{av} = t_{av}(\tau)$.

5. We plot the curve for the change in the temperature of the metal, for the case in which $r = r_1$, on the basis of the formula

$$t_1(\tau) = t_{av}(\tau) + \frac{q(\tau)}{\beta E}(1-\mu) S_{1\infty}. \tag{11}$$

6. By means of the solution for the system of equations (1) and (11) for T at various instants of time, we plot the sought curve for $T = T(\tau)$.

Let us consider an example. In a conduit made of 15Kh1M1F steel with dimensions $2r_1 = 0.155$ m and $2r_2 = 0.245$ m, and with an initial temperature of t = 100°C we have to raise the temperature of the steam from 100 to 300°C to feed it into the turbine. With a change in the temperature of the steam at a constant rate the permissible value of $dT/d\tau = 0.08$ deg C/sec, $[\sigma]_1 = 2.1 \cdot 10^7$ N/m, and $\alpha = 116.3$ W/m·deg. We see from the curves in Fig. 2 that because of the smallness of the selected value of α – corresponding to actuation of the unit with sliding parameters – in the case under consideration, with use of the proposed method, the time for the heating of the conduit can be reduced by a factor of 10.

NOTATION

τ	is the time;
Fo	is the Fourier number;
r	is a coordinate;
T	is the temperature of the heating medium;
t	is the conduit temperature;
$t_{av} = 2\int_{r_1}^{r_2} tr dr / r_2^2 - r_1^2$;	
q	is the density of the heat flow through the inside surface of the conduit;
σ	is the permissible stress;
σ_φ, σ_z, and σ_r	are the tangential, axial, and radial stresses;
$S = \sigma_\varphi$	for q = 1;
$S_\infty = S$	when $\tau = \infty$;
E	is the modulus of elasticity;
β	is the coefficient of linear expansion;
μ	is the Poisson coefficient;
a	is the coefficient of thermal diffusivity;
α	is the heat-transfer coefficient;
$0 < \theta < 1$;	
$I(b, c) = \int_b^c (dq/d\tau_0) S(r, \tau - \tau_0) d\tau_0$.	

Subscripts and Superscripts

1 is the inside surface;

2 is the outside surface.

LITERATURE CITED

1. A. V. Luikov, The Theory of Heat Conduction [in Russian], Vysshaya Shkola (1967).
2. W. Endres, Brown Boveri Mitteilungen, 45, No. 1, 21-28 (1958).
3. D. L. Kostovetskii, B. N. Tokarskii, and N. V. Makarov, Elektricheskie Stantsii, No. 6, 2-6 (1968).
4. S. D. Ponomarev (editor), in: Strength Calculations in Machine Building [in Russian], Vol. 2, Mashgiz (1958).